

## Designing Mathematics Model of Teaching: The syntax of “Problem-Solving Performance Modelling” Model of Teaching

Nurcholif Diah Sri Lestari  
[nurcholifdsl@yahoo.com](mailto:nurcholifdsl@yahoo.com)

Titik Sugiarti  
[titiksugiarti.fkip@unej.ac.id](mailto:titiksugiarti.fkip@unej.ac.id)

Faculty of Teacher and Training Education, Jember University, Indonesia

### Abstract

The implementation of 2013 curriculum has come under spotlight in these recent years. The curriculum expands the competences into four aspects. They are spiritual attitudes, social attitudes, knowledge, and skills. The curriculum also promote the use of scientific approach, problem solving and problem based learning in the every teaching and learning process. Unfortunately, based on the initial interviews amongst teachers, it was founded that problem solving is rarely used in teaching mathematics. This is due to the difficulty in teaching and assessing problem solving in the same time. Therefore, students' ability to solve the problem is not trained well.

This paper attempt to present the syntax of a new model of teaching as a part results of continuation research of the first year research. The model is designed to teach and to assess the problem solving skill in the same time through a modelling performance. Modelling performance is refers to modelling about how students problem solution (presented through exemplars problem) is assessed using authentic assessment by exemplars rubric. For this purpose, a developmental research based on Plomp (1997) was designed in order to get the mathematics model of teaching to develop students' ability in problem solving. The research conducted in four steps, i.e. preliminary investigation, design, realization, and test, evaluation and revision. The third first steps were conducted in first year research. By the first year research, it was gained the model (syntax) of teaching in theoretical form. The last step of plomps' is conducted in the second year research. The last step is conducted by expert validation, test the model in the real class and then revise it if needed.

The designed model of teaching is called as Problem-Solving Performance Modelling. The model consists of eight phases. The phases begin by the orientation, individual problem solving (using exemplar problem), organizing the group discussion, group discussion, classical discussion, modelling of problem solving performance (using exemplar rubric), evaluating, and closing.

**Keyword:** problem solving, performance modelling, model of teaching

### 1. Introduction

Students of elementary school possess a great amount of curiosity toward anything. According to Ernst in Kallick and Brewer (1997:12-13), this characteristic is one of attitudes which has to be owned by a good problem solver. The attitude can be greatly exploited by the teachers' through problem solving as a medium to teach students about reasoning, critical and creative. By then, the learning process will be more attractive.

Unfortunately, problem solving was not something that could be easily applied. Many teachers at school preferred to teach mathematics by applying memorization technique in dealing with routine problem. That circumstance often took place in teaching and learning process in the elementary schools throughout Indonesia.

In an informal interview between the writer and an elementary school teacher, it was found out that problem solving strategy was once applied in class, yet it was run only for a while. Most of the teachers faced distinctive barriers regarding: (1) how to teach problem solving competence without overlooked the teaching goals and, (2) how to assess problem solving skill such that the assessment is focused not only on the final product but also on the process in problem solving and suitable for teaching goals, (3) how to develop student's problem solving skill. Therefore, a research to develop new models of teaching is needed.

## 2. Exemplar Problem and Exemplar Rubric

Kallick and Brewer (1997) was introduced "exemplars problem" and "exemplars rubric" as a tools to teach problem solving to K-2 students in California. Exemplars Problem consisted of a series of pieces of mathematics problem. Based on Kallick and Brewer statements, so in this teaching model, the exercises used was uncued world problem. That was incomplete problem in daily life or open ended that could not indicate the clear instruction to solve, for example, there were some incomplete missing part. Therefore, those problems had some solutions (open ended). There was a different specific mathematical problem on each piece of paper. In this model of teaching, the exemplar problem was given continually for once a week at most for elementary school students. This frequency was different to what Kallick and Brewer (1997) oppinion, that was done at most twice in a week. It was caused that the learning process in Curriculum 2013 was implemented based on integrative thematic (Depdiknas, 2013). All subjects provided in the form of themes. Every theme consisted of some subthemes and each subtheme was taught for a week. Thus, the knowledge was comprehensively dealing with the implementation of mathematics would obtain the end of one subtheme (a week).

Exemplar rubric is a kind of authentic assessment to assess students' ability in solving the problem that written in the exemplar problem. Kallick and Brewer (1997), introduced exemplar rubric that was arranged based on the levels of students' ability (differ in four levels). The level is described by three criteria, i.e: (1) understanding (2) strategy, reasoning, and procedure (3) communication. The criteria for each level is differ by the indicators. Then, in this model of teaching the indicators has been modified. The modifications were based on the elementary school students' situation and condition in Indonesia.

There were two kinds of exemplar rubric, exemplar rubric for teachers and exemplar rubric for students. The content of both were arranged well in the same meaning. For students, it was provided simpler in language and reflected to what was intended to do. This rubric was not only used as an assessment but also as the part of teaching learning process. By using the exemplar rubric, hopefully, the students can improve their ability to construct the thinking in problem solving, to know what they need in checking the accuracy, and the quality of their works. Thus, the students are able to assess their works before submitting to the teacher.

## 3. Frameworks

Commonly, the teaching learning mathematics process demands the students' procedural ability. The observation result that has been done by the researcher (in introduction) showed that the students were usually taught certain concept by

memorizing concept or some formulas. After that, the teacher gave the exercise based on the book (usually they use only one book). The chosen questions in this book were the exercises that have a correct answer and often times it was not a problem. The teacher considered that mathematics was a product that was ready to be transferred to the students. They rarely teach problem solving because the difficulties in teaching and assessing.

To make students ready with problem solving, teacher should introduce variety of strategies to solve a problem before the learning process was executed. Some variety of problems with different strategy also must be provided to enrich the strategy and to improve the ability. It is supported by Lestari's research result that the more often students encountered a problem, the more competent they would be in solving a problem (2010). While the problem solving skill is taught then the assessment also can be held. In this learning model, the students were taught to observe how their results of problem solving were assessed through exemplars rubric. By then, they could determine the purpose and the direction dealing with what they should do to achieve the best score in problem solving.

"Problem Solving Performance Modeling" model of teaching is a learning model that provides the problem solving items (through exemplar problem) and the expected performance (through exemplar rubric). One of the phases of this model was group discussion to solve the problem. The group discussion would train the students to interact with another student. Moreover, by the interaction, the students would see the method or the way that was found or suggested to them in their groups. This activity was expected to be facilitated well.

On the other hand, every student was expected to be active in learning process. By the exemplar problem and exemplar rubric, the students have been given the opportunity by the teacher to find the solution of the problem and to measure their ability in problem solving by themselves. As the result, the students have had at least the experience about how to solve the problem. Furthermore, this activity could help the students to do some metacognition process

According to the explanation above, it indicates that "problem solving performance modeling" model of teaching could train the students to enhance the problem solving skill rather than the traditional teaching method asking a correct answer. By using "problem-solving performance modeling" model of teaching the students' interaction with others could be improved by the group discussion, and the communication ability could be also improved through the presentation.

#### 4. Research Methodology

The research was a developmental research based on the design research model by Tjeerd Plomp (1997). Plomp used four phases in the development. They were initial investigation, design, realization, and the last phase consists of test, evaluation, and revision. The research was held in two years. In the first year the research (Sugiarti and Lestari, 2013) was aimed to design a framework of the model of teaching syntax (phase one until three) such that the problem solving skill could be taught to the student and assessed in one time. While the second year research is aimed to evaluate, test and revise the syntax model of teaching. The phases used in this research are:

- a. **The Phase of Initial Investigation.** In this phase, there were study literatures activities and discussions between researchers on (1) Relevant learning theories, (2)

Relevant strategy or method of teaching, (3) Finding the devise to assess and teach problem solving, (4) Authentic assessment, (5) the 2013 curriculum.

- b. The Phase of Design.** During this phase, based on the initial investigation phase then the framework of mathematics model of teaching was designed. The activities in this phase were devise the frameworks of (1) learning environment or social system, (2) principles of reaction, (3) supporting system, (4) the learning impact, (5) the learning syntax.
- c. The Phase of Realization.** This phase is held as a continuation from the design phase. The activities solely conducted during this phase cover: (1) determining learning environment or social system, (2) structuring reaction principle, (3) determining supporting system, (4) structuring learning impacts, (5) constructing learning syntax. Teaching model resulting from the current phase is then called as prototype I.
- d. The Phase of Test, Evaluation, and Revision.** During the first year of the research, the research merely focuses on the derivation of valid model of teaching. In order to validate the model, there are numerous activities which have to be done, covering: (1) asking for expert and teacher's consideration on the expediency of the structured teaching model prototype, (2) conducting analysis toward the validation result of validators. If the analysis outcome indicates invalid result then the prototype must be revised. Therefore in this phase, the process may become a cycle in validation activity.

## 5. Result

By the methodological research then the syntax of "problem-solving performance modeling" model of teaching were developed. The syntax are listed bellow:

### Pre-learning

In this activity, the teacher gave pretest in the form of problem solving test to the students. Then, the results were assessed and analyzed based on the teacher's exemplar rubric to categorize the students in the level of problem solving skill according to Kallick & Brewer (1997). Then, the level of students' ability became the scores or the first level that would be assessed how far the improvement happened after teaching learning process.

### Phase 1. Orientation

The purpose of this phase was to make the students were ready to learn. Thus, the phase was a necessary part in teaching learning process. In this phase, the teacher had to tell the learning objective and motivate the students through presenting the examples of question related to daily life. As the result, the students would be motivated, interested, and focused on learning. Then, the teacher needed to refresh the materials or the strategies of problem solving that was used to solve the problems. Therefore, the students were more ready to face the given problems. More strategies given, more ability improved to solve the problems. Besides providing the strategy variation, the teacher also gave the students some experiences about how to assed their solution through the examples of student's solution on pre-test and the exemplar rubric filled. The next activity was to inform how the students were assessed. When the students known how they would be assessed so they would be more ready to conduct

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the assessment. While this activity was implemented, the students had to pay attention on the demonstration given by the teacher and asked some questions.

### **Phase 2. Individual Problem Solving**

In this phase, the learning has been transformed from teacher centered into student centered. After the teacher distributed exemplar problem (had been prepared), the guidelines of problem solving (had been prepared), and the exemplar rubric. The teacher should explain how the guidelines could be used to help them solve the problem and how the rubric help them to reach the expected ability in problem solving. Thus, the students began to do exemplar problem individually. Moreover, the characteristics of the uncued problem causing multi answer or multi methods to solve. It also could develop the students' ability and creativity in solving the problem. In this phase, the teacher began to do the research, note taking, and assess the students' affective during the learning process using affective assessment rubric (prepared as the teaching purposes).

### **Phase 3. Group Organization**

In the third phase, the students were placed in cooperative learning. They were placed into some heterogeneous group based on the students' level of solving problems (the pretest result on pre-learning or the students' previous result). Every group consisted of 4-5 students having the different level of problem solving.

### **Phase 4. Group Discussion**

The purpose of this phase was to improve the reasoning in solving the problems through sharing in a group discussion. The groups were given the assignment to discuss the exemplar problem that has been done individually in the form of students' work sheet (LKS). In this activity, every student shared their ideas in their group and determine the easiest and the most effective way to solve the problems based on the instruction in LKS. Thus, through group discussion phase, the strategy of problem solving could be enriched by sharing their learning experience to others in a group so their abilities were improved. On the other hand, the scaffolding principle has been done. When the students discussed in group, the teacher became a facilitator guiding them to do LKS well. The teacher also conducted observation and affective assessment during learning process.

### **Phase 5. Classroom Discussion**

In this phase, the teacher asked the representatives of some groups to present their result. When the representative explained the result in front of the class, the other groups had to give suggestions and arguments. The member of presenting group also helped each other. In classroom discussion, the role of the teacher was moderator and facilitator relating the presenting group to the audience, and made the discussion atmosphere well. Through the classroom discussion, the students were gotten the learning experience in problem solving more deeply. They would learn how the best way to solve the problems or found various solutions based on the other groups' point of view. Thus, the students were more confidence to give their opinion because actually, the various problem solving was not definitely false.

### **Phase 6. Assessment Modelling**

After presenting and responding the results, the teacher gave the example of how the results were assessed using exemplar rubric in the class. Based on the assessment example done by the teacher, every student was asked to assess their result by their own individually. This activity made the students more critic and logic to solve the problem because they could determine which was done or not.



### Phase 7. Evaluation

The evaluation phase was intended to examine the students' ability to solve the problems after teaching learning process. The teacher distributed another exemplar problems and the guidance of problem solving. The students were asked to solve the problems in exemplar problem based on the guidelines and submit their work to the teacher.

### Phase 8. Closing.

The phase was intended to review and combine the knowledge gained by students. This review could be done by some ways such as by giving some questions that encourage the students to achieve the point of their learning.

### Post-Learning

Learning by using authentic assessment through exemplar problem could be done repeatedly (cycle) using different exemplar problem (especially dealing with the strategy of problem solving used) so the students got another way of problem solving. Thus, they will be better to do the uncued mathematics problem. The learning using this model was suggested to implement at most once a week. The final activity was giving post test to the students to measure their level of problem solving after teaching learning process was done. Table 1 below showed how the teacher and student activity dealing with the syntax learning using the syntax of "Problem-Solving Performance Modelling" model of teaching

Table 1. The syntax in Mathematics Model of Teaching Based on Authentic Assessment through Exemplars Problem

No	Phase	Teacher's Activities	Students' Activities
	Pre Teaching	Categorizing the level of the students' problem solving skill by conducting pre test	Doing the pre test
1	Orientation	<ul style="list-style-type: none"> <li>• Present the learning objective and motivating students through the provision of problem samples closely related to daily life.</li> <li>• Pointing out explanation or review on the prerequisite materials and or problem solving strategies which might be possibly applied to solve problem in the next phase</li> <li>• Distributing exemplars problem and exemplars rubric that fulfilled and assessed</li> <li>• Explain how to use the exemplar rubrics and show how exemplar problem would be assessed</li> </ul>	<ul style="list-style-type: none"> <li>• Listening to teacher's explanation, answering or solving the given problem</li> <li>• Listening to teacher's explanation, making notes, receiving the exemplars problem and exemplars rubric, asking when there is explanation in question, and responding to teacher's inquiries.</li> </ul>
2	Individual Problem Solving	<ul style="list-style-type: none"> <li>• Distributing the exemplar problem and the guide of problem solving</li> <li>• Explaining how to use the guide of problem solving</li> <li>• Asking the students to do the exemplar problem individually</li> <li>• Observing, making a note and assessing on how students solve a problem</li> </ul>	<ul style="list-style-type: none"> <li>• Receiving exemplar problem and guide of problem solving</li> <li>• Listen to the teacher's explanation and ask questions if find the miss understanding</li> <li>• Work on exemplars problem by referring to the exemplars rubric. In solving the problems, they can do the problem solving on the exemplars rubric directly.</li> </ul>
3	Group Organization	<ul style="list-style-type: none"> <li>• Organizing students into heterogeneous learning groups based on their problem</li> </ul>	<ul style="list-style-type: none"> <li>• Deploying themselves into their group</li> </ul>

		solving level (4-5 students).	
		<ul style="list-style-type: none"> <li>• Distribute worksheet (exemplars problem and exemplar rubric which are exactly similar to the previous) for each group.</li> </ul>	<ul style="list-style-type: none"> <li>• Receiving, read and try to understand the exemplars problem and exemplars rubric.</li> </ul>
4	Group Discussion	<ul style="list-style-type: none"> <li>• Asking the students to do the worksheet</li> <li>• Ask the student to share idea to repair their work in order to find the best solution</li> <li>• Observing, making note, and assessing the student attitude in solving a problem .</li> <li>• Being a facilitator in group discussion and giving help to groups if they find any difficulties.</li> <li>• Ask the students to collect the work discussed</li> </ul>	<ul style="list-style-type: none"> <li>• Doing the worksheet</li> <li>• Sharing ideas in groups when working on exemplars problem by referring to problem solving guidance</li> <li>• Asking a question if they have any difficulties</li> <li>• Collect the discussion result</li> </ul>
5	Class Discussion	<ul style="list-style-type: none"> <li>• Ask the representatives from several groups to present their discussion result.</li> <li>• Facilitating the class discussion refer to the exemplar rubrics</li> <li>• Observing, making note, and holding assessment the student in both affective and cognitive aspect of problem solving</li> </ul>	<ul style="list-style-type: none"> <li>• Some group present their discussion result.</li> <li>• The other students from other groups have to comment or ask some questions on the presentation by refering to exemplar rubrics or problem solving guidance</li> </ul>
6	Assessment Modelling	<ul style="list-style-type: none"> <li>• Demonstrate on how the groups solve problem</li> <li>• Asking the students to assess their individual problem solving</li> </ul>	<ul style="list-style-type: none"> <li>• Listening to teacher's explanation, making notes, receiving exemplars rubric and exemplars problem,</li> <li>• raising questions if there are any question.</li> </ul>
7	Evaluation	<ul style="list-style-type: none"> <li>• Evaluating the students achievement</li> </ul>	<ul style="list-style-type: none"> <li>• Doing the evaluation</li> </ul>
8	Closing	<ul style="list-style-type: none"> <li>• Both teacher and students cooperatively infer ideas or concepts that have been elaborated in that day. The techniques applied can be greatly varied. Such technique as teacher's giving inquiries which stimulate them to reach the desired points</li> </ul>	
	Post Learning	Giving the post test and categorizing students based on their ability	Making notes on the elaborated conclusion

## 6. Discussion

Similar to another learning model that have specific characteristics so “problem-solving performance modeling” model of teaching also has specific characteristics. Therefore, some preparation is needed before implementing this learning model.

The first is planning. The activities that must be done by the teacher is formulating the learning objectives, determining the materials, designing, preparing authentic assessment instrument, conducting pre-test to identify the students' ability in problem solving using exemplar problem and teachers' exemplar rubric, constructing the heterogenic groups, conducting post test and placing the students based on their ability of problem solving. It is done to know is the teaching learning process can improve the students' ability. The materials chosen can be in the form of thematic materials. This model can be used to construct a concept as far as the students have the materials before.

The second is learning management. The learning management using “problem solving performance modeling” model of teaching was done based on its syntax. Based on the explanation above, this model has pre-learning phases. The syntaxes are orientation, group organizational, problem solving individually, group discussion, classroom discussion,, assessment given, evaluation, closure, and post-learning phase.

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The teacher and students must be active during teaching learning process. In elementary school level, the time allocation suggested is a day ( $5 \times 35$  minutes)

The third is assessment. The assessment in “problem solving performance modeling” model of teaching is done holistically dealing with cognitive assessment, psychomotor, and affective. The assessments of cognitive and psychomotor are done through exemplar problem and exemplar rubric. The mapping between the cognitive and psychomotor can be viewed in exemplar rubric for the teacher. The affective assessment is gained during the teaching learning process.

### **Acknowledgements.**

The author would like to thank the Indonesia’s DIKTI government that have given the grant for us to do the research .

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